

**3.2. OPERATIONAL USE OF MATHEMATICAL  
MODELLING**

**Doug Chambers  
Senes Consultants Ltd.**



# **Operational Use of Mathematical Modelling Prediction of Acid Mine Drainage**

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## **Outline**

- ◆ **Site Assessment and Data Review**
- ◆ **Data Collection, Sampling, Testwork**
- ◆ **Model Selection or Model Development**
- ◆ **Model Application**
- ◆ **Interpretation**

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## **Site Assessment and Data Review**

- ◆ Site Visit, Photographs
- ◆ Site Plans, maps, ....etc.
- ◆ Monitoring Data (exploration, development and operation)
- ◆ Laboratory Analyses, Testwork, Field Studies
- ◆ Mine Plan (development, operation, decommissioning)

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## **Data Collection, Sampling and Testwork**

- ◆ Design Site-Specific Program
- ◆ Select Protocols for Sampling and Testwork
- ◆ Apply Reference Handbooks such as:
  - Waste rock sampling manual (MEND 4.5.1)
  - Environmental monitoring handbook (MEND)

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## **Why Model Tailings and Waste Rock Heaps ?**

- ◆ To Examine Current State Of Acid Generation
- ◆ To Predict Future Contaminant Loads and Concentration Profiles
- ◆ To Assess Future Treatment Requirements
- ◆ To Compare Management and Decommissioning Options

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## **Mathematical Approach**

- ◆ Empirical or Statistically-Based
  - Site water-quality database
  - Laboratory testwork
- ◆ Mechanistic or Based on Theoretical Equations
  - Kinetics of sulphide oxidation
  - Oxygen transport
  - Thermodynamic equilibrium
  - Mass transport

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## **Data Requirements for Geochemical Modelling (1)**

### **◆ Climatic:**

- Monthly mean surface temperature
- Monthly precipitation / evaporation/  
evapotranspiration

### **◆ Physical:**

- Surface area, depth
- Bulk density, porosity / void volume
- Moisture content / saturation level (% void volume)
- Particle size distribution

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## **Data Requirements for Geochemical Modelling (2)**

### **◆ Hydrogeological:**

- Hydraulic conductivity (vertical & horizontal)
- Infiltration flow
- Seepage flow

### **◆ Water Quality:**

- Infiltration waters
- Background water quality (surface, groundwater)
- Porewater quality (with depth)
- Seepage Quality

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## **Data Requirements for Geochemical Modelling (3)**

### **◆ Mineralogical:**

- Mineralogical composition
- Estimate of percent leachable pyrite (sulphide minerals)
- Chemical and elemental analyses of solids
- Chemical formulae of minerals
- Proportion pre-oxidized (easily soluble) metals and radionuclides

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## **Data Requirements for Geochemical Modelling (4)**

### **◆ Kinetic:**

- Biological and chemical oxidation rates for sulphide minerals
- Weathering rates for silicates and carbonates

### **◆ Miscellaneous Data:**

- Description of historic as well as current activities
- Visual observations reported during field visits

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## **Data Requirements for Geochemical Modelling (5)**

- ◆ **Transport Related Properties:**
  - Diffusion coefficients
  - Convective air flow
  - Thermal conductivity
  - Temperature measurements with depth
  - Solid-liquid distribution coefficients ( $K_d$ ) for metals and radionuclides

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## **Some Of The Major Uncertainties (1)**

- ◆ **Mineralogy:**
  - Sulphide minerals (% fines)
  - Buffering minerals (carbonates and aluminosilicates)
  - Secondary minerals (stored acidity, sorption)
- ◆ **Proportion of Easily Leachable  
(pre-oxidized) Metals and Radionuclides**

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## **Some Of The Major Uncertainties (2)**

- ◆ **Flows and Quality (infiltration, porewater, seepage)**
- ◆ **Porosity and Moisture Content**
- ◆ **Oxygen Transport (convection, diffusion)**

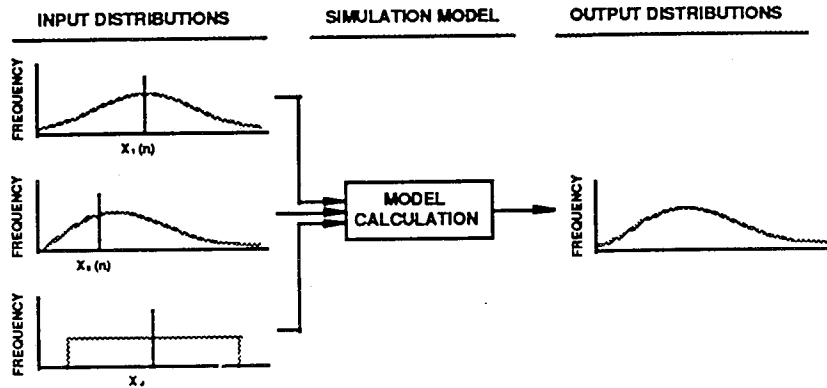
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## **How to Deal with Uncertainty ?**

- ◆ **Perform Deterministic Sensitivity Analyses to Examine Influence of Particular Parameters or Assumptions**
- ◆ **Employ Distributed (probabilistic) Parameter Values**
- ◆ **Perform Preliminary Investigative Modelling (eg. empirical or physical modelling followed by geochemical)**

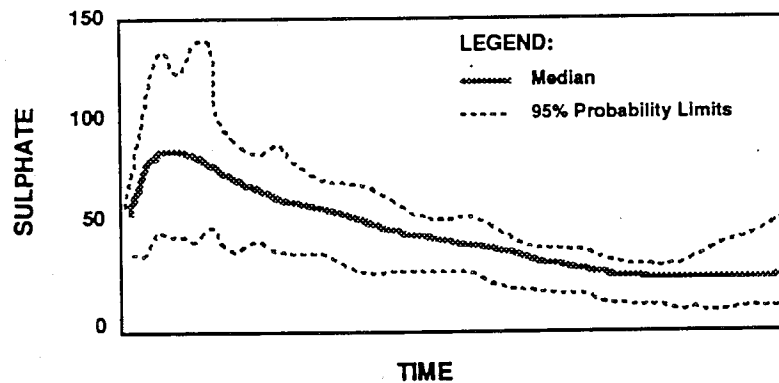
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# Concept of Probabilistic Analysis



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# Defining Uncertainties



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## **Empirical Prediction (1)**

- ◆ **Examine Water Chemistry Database**
  - Adequate data
  - Detection limits
  - Remove outliers
- ◆ **Compile Summary Statistics (aqueous, solids)**
- ◆ **Uni or Bivariate Techniques**
  - Time plots
  - Correlation analyses
  - Regression analyses

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## **Empirical Prediction (2)**

- ◆ **Multivariate Techniques**
  - Principle component analyses
  - Cluster, factor, discriminant, etc.
- ◆ **Assess Cyclical or Repeating Trends**
  - Time series modelling
- ◆ **Develop a Model**
  - Cross-validation
  - Assess confidence or reliability

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## Parameters and Elements Correlation Matrix for Aqueous Species

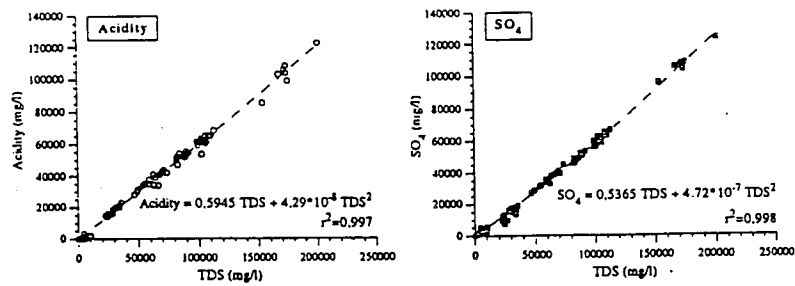
	Fe <sub>tot</sub>	Acidity	TDS	SO <sub>4</sub>	Conduc	pH	Ca	Al	SG*	Mg	Fe <sup>2+</sup>	Fe <sup>3+</sup>	Et
Fe <sub>tot</sub>	1												
Acidity	.971	1											
TDS	.978	.997	1										
SO <sub>4</sub>	.971	.994	.998	1									
Conduct	.951	.958	.964	.959	1								
pH	-.409	-.402	-.377	-.354	-.380	1							
Ca	.734	.700	.723	.725	.804	-.257	1						
Al	.923	.979	.977	.977	.929	-.328	.663	1					
SG*	.971	.988	.998	.997	.949	-.855	.956	.977	1				
Mg	.931	.964	.970	.973	.937	-.233	.697	.965	.991	1			
Fe <sup>2+</sup>	.800	.799	.819	.828	.819	-.029	.655	.822	.636	.876	1		
Fe <sup>3+</sup>	.683	.636	.624	.604	.579	-.642	.419	.530	.936	.476	.108	1	
Et	-.182	-.191	-.220	-.227	-.266	-.546	-.402	-.239	-.578	-.324	-.440	-.235	1
log(Fe <sup>2+</sup> /Fe <sup>3+</sup> )	.345	.353	.382	.392	.437	.429	.553	.404	.637	.477	.465	-.213	-.917

\*Specific Gravity: only 15/93 samples.

MEND Report 1.14.2, Monitoring of Acid Mine Drainage:  
Chemical Data from Mine Dayon South Waste Rock Dump  
(OREQI 85-04, Université Laval)

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## Relationships Between TDS and Major Elements (TDS >25,000 mg/L)

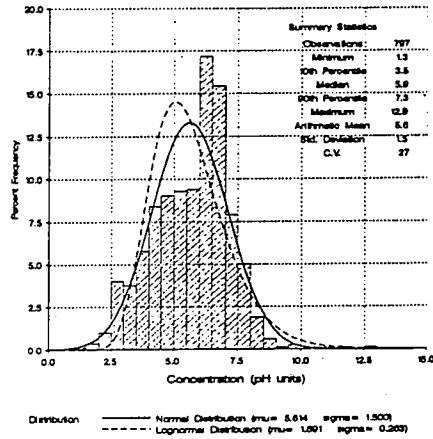


**Note:**  
Used to reduce sampling frequency (costs)

MEND Report 1.14.2, Monitoring of Acid Mine Drainage:  
Chemical Data from Mine Dayon South Waste Rock Dump  
(OREQI 85-04, Université Laval)

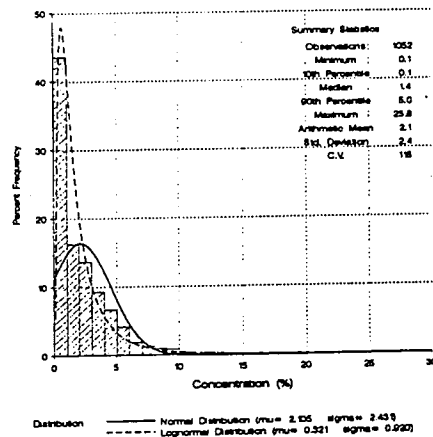
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# Absetzerhalde Paste pH



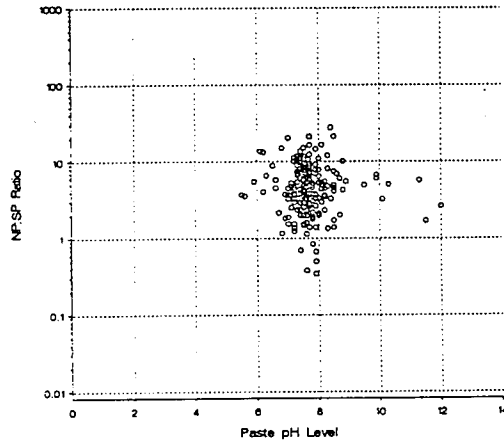
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# Absetzerhalde Percent CO<sub>2</sub>



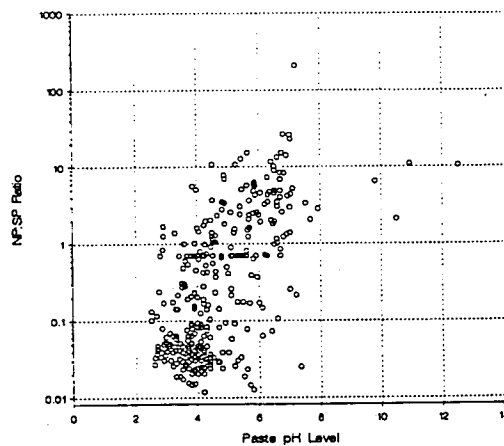
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## Drosen NP / SP Ratio vs. Paste pH



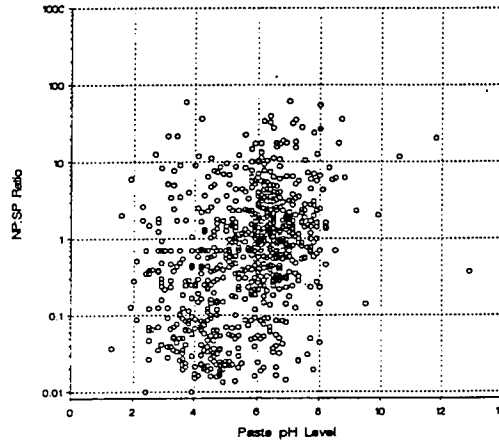
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## Nordhalde NP / SP Ratio vs. Paste pH



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## Absetzerhalde NP / SP Ratio vs. Paste pH



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## Cross Tabulation of NP:SP vs. NNP (Absetzerhalde)

Frequency Row Percent Column Percent	NNP Subset			Total
	NNP < -20 (Acid Generating)	-20 ≤ NNP < 20 (Uncertain)	NNP ≥ 20 (Non-acid Generating)	
NP:SP < 1 (Acid Generating)	434 68.24 100.00	202 31.76 88.99	0 0.00 0.00	636
1 ≤ NP:SP < 3 (Uncertain)	0 0.00 0.00	25 10.00 11.01	225 90.00 57.54	250
NP:SP ≥ 3 (Non-acid Generating)	0 0.00 0.00	0 0.00 0.00	166 100.00 42.46	166
<b>Total</b>	<b>434</b>	<b>227</b>	<b>391</b>	<b>1052</b>

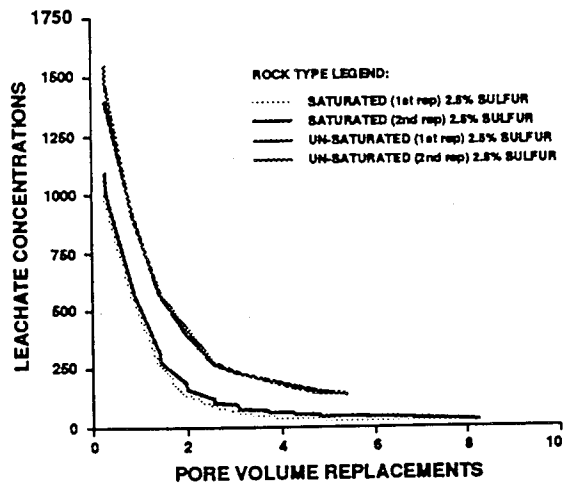
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## Empirical Prediction of Future Water Quality, SENES

- ◆ Analyse Results Obtained from Kinetic Testwork (e.g. humidity cell, columns)
- ◆ Develop an Empirical Expression Relating Constituent Concentration to Porewater Replacement Volume
- ◆ Apply this Expression Within a Mechanistic Framework (e.g. flow and mass transfer model)
- ◆ Perform an Environmental Pathways Assessment

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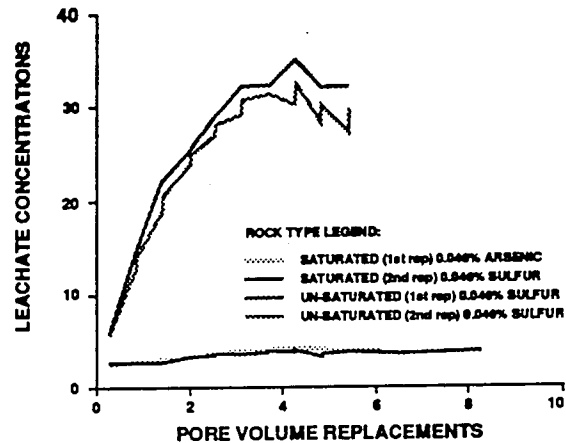
## Sulfate Concentrations (mg/L) in Leachate



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## Arsenic Concentrations (mg/L) in Leachate



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## Empirical Prediction

- ◆ Preliminary or “Order of Magnitude” Estimates of Annual Acid Generation Based on One Primary Factor:
  - Oxygen transport (diffusion)
  - Infiltration flow
  - Example → Equity Silver

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## Estimates of Peak ARD Rates (t/a) for Main Dump and Bessemer Dump

	PEAK MEASURED (1990)	OXYGEN DIFFUSION	INFILTRATION	TECHNICAL COMMITTEE	MODEL RESULTS
No Cover	-	30,000	16,500	-	-
Loose Till Cover (1.4m)	8,000	12,500	11,000	17,100	7,800
Non-Compacted Clay (0.7m)	-	14,575	6,125	-	*
Semi-Compacted Clay (0.7m)	-	7,600	3,750	12,100	*
Compacted Clay (0.7m)	-	430	280	-	*

**NOTE:**

Peak Occurs Before Placement of Cover \*

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## Important Processes in Geochemical Modelling (1)

- ◆ Acid Generation Due to Biological and Chemical Oxidation of Sulphide Minerals
- ◆ Diffusion of Oxygen into Heaps and Tailings, and also into Tailings Rock Particles
- ◆ Convective Transport of Oxygen into Waste Rock Heaps

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## **Important Processes in Geochemical Modelling (2)**

- ◆ **Production and Transport of Heat (conduction and advection)**
- ◆ **Temperature With Depth and Effect of Temperature on Oxidation Rates**
- ◆ **Release (Leaching) of Metals and Radionuclides**

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## **Important Processes in Geochemical Modelling (3)**

- ◆ **Transport of Dissolved Chemical Species (eg. sulphate, aluminium, calcium, magnesium, iron, metals, radionuclides)**
- ◆ **Dissolution of Solid Buffering Minerals (eg. calcite, dolomite, sericite)**
- ◆ **Formation (precipitation) of Secondary Minerals (eg .ferric hydroxide, jarosite)**

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## Important Processes in Geochemical Modelling (4)

- ◆ Solubility of Solid Phases  
(eg. precipitates, minerals)
- ◆ Speciation of Dissolved Constituents
- ◆ Estimation of the pH

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## Order of Selection for Buffering Minerals

pH	Minerals
> 6.2	Calcite
> 5.4	Siderite
> 4.8	Aluminium Hydroxide Magnesium Oxide
> 3.1	Ferric Hydroxide
< 3.1	None or Jarosite

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## **Important Processes in Geochemical Modelling (5)**

- ◆ **Solids Solution Equilibria for Metals and Radionuclides**
- ◆ **Adsorption of Metals and Radionuclides onto Aluminium and Ferric Hydroxides, Jarosite and Organic (Carbon) Surface**
- ◆ **Co-precipitation (e.g.. radium with gypsum, lead-210 with lead sulphate, lead carbonate ... etc.)**

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## **General Approach to Mathematical Predictions (1)**

- ◆ **Identify Objectives**
- ◆ **Collect and Review Site Data**
- ◆ **Select, Adapt or Develop Model(s)**
- ◆ **Prepare Model Inputs and Parameter Estimates**
- ◆ **Calibrate Model to Field Data**

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## General Approach to Mathematical Predictions (2)

- ◆ Perform Simulations
- ◆ Interpret the Results
  - Identify controlling processes
  - Compare to concentrations at other sites
  - Compare to estimates obtained using alternate approach

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## Overview of Models (1)

Types of Models	Typical Examples	SENES Models	Applications of SENES Models
Thermodynamic Equilibrium Models	MINTEQ	PHCALC	- water quality analyses / addition of alkaline amendments - treatment ponds
	PHREEQE	GOLDTAIL	
Mass Transfer Models	EQ6 PATHARC		- see below
Coupled Mass Transfer Models	MINTRAN	PITMOD	- mine wastes placed in flooded pits - source term module within environmental pathways models
	PHREEQM	UTAP	
		Site-specific empirical models	- kinetic experiments (eg columns, humidity cells)

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## Overview of Models (2)

Types of Models	Typical Examples	SENES Models	Applications of SENES Models
Support or Physical Models (eg air/heat transport)	FIDHELM TOUGH AMD	CONVECT MINEVENT ROCKPIT	- waste rock heaps - underground mines - backfilled pit mines
Engineering Models	Q-ROCK MINEWALL WATAIL MINTOX RATAP	ACIDROCK ROCKSTAR URANTAIL	- Equity Silver heaps, Gessenhalde In-situ heap leach, coal spoil piles - Ronneburg heaps, Ronneburg mining region - gold, nickel, zinc and uranium tailings sites

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## CONVECT and ROCKPIT (1)

- ◆ Steady State Physical (computer) Models
- ◆ Transport of Oxygen and Heat
- ◆ Waste Rock or Tailings
- ◆ CONVECT
  - Pyramidal Coordinates
- ◆ ROCKPIT
  - Cylindrical Coordinates

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## **CONVECT and ROCKPIT (2)**

- ◆ **Depth Profiles for Oxygen, Temperature and Pyrite**
- ◆ **Diffusion or Convection ?**
- ◆ **Estimates of Convective Airflow Rates**

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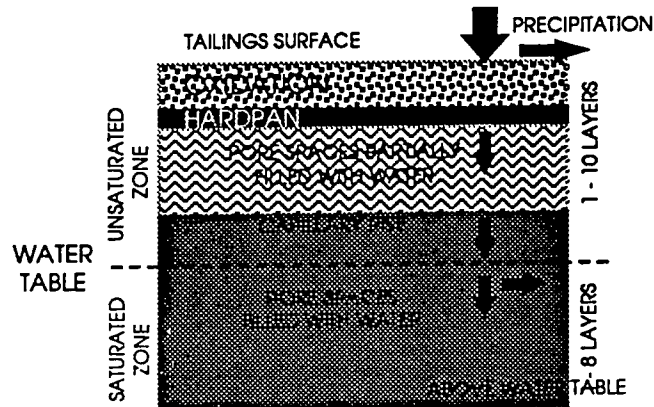
## **URANTAIL**

- ◆ **Dynamic Geochemical (computer) Model**
- ◆ **Acid Generating or Alkaline Tailings**
- ◆ **Gold, Base Metal, Uranium Tailings**
- ◆ **Wet or Dry Closure Alternatives**
- ◆ **Flooded (pond) With/Without Overflow or Recycle**
- ◆ **Simple to Complex Engineered Covers**
- ◆ **Predictions of Long-Term Water Quality**

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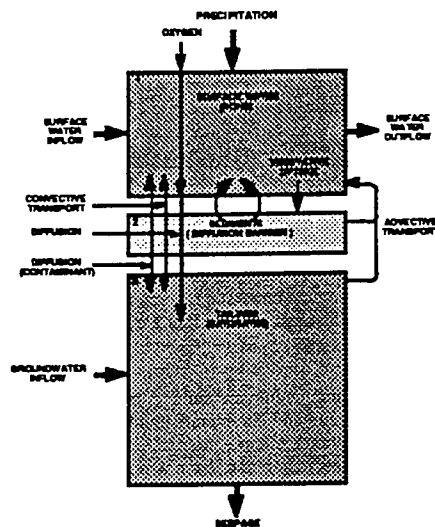


## Concept for Modelling Tailings



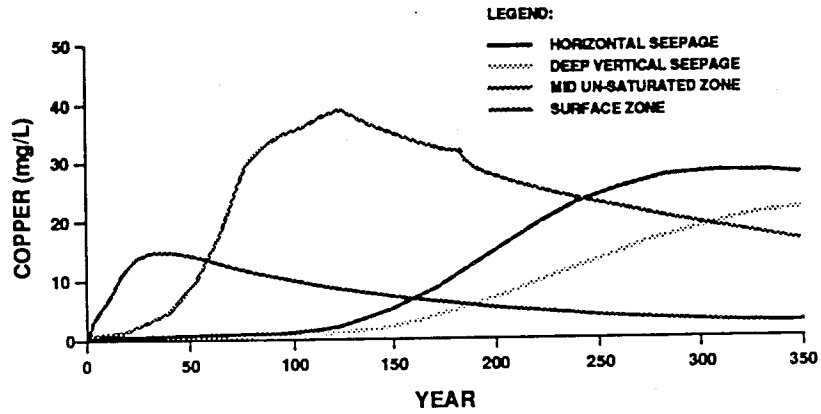
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## Concept for Modelling Submerged Tailings



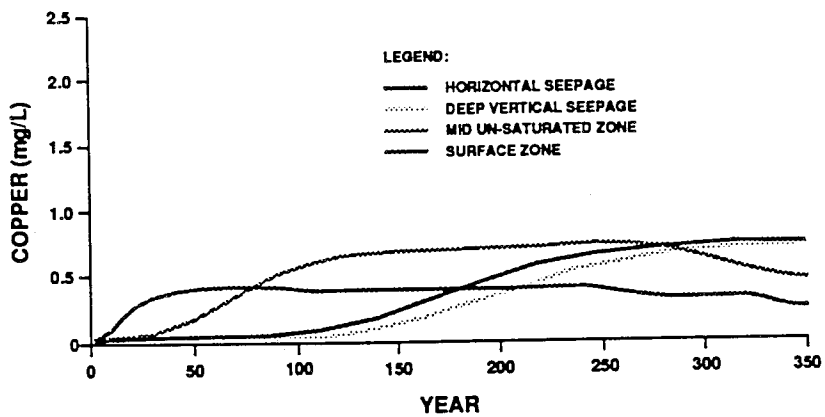
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## Simple Cover on Coarse (Gold) Tailings



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## Engineered Cover on Coarse (Gold) Tailings



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## **ACIDROCK (1)**

- ◆ **Dynamic Geochemical (computer) Model**
- ◆ **Waste Rock Heap to Fines and Rock Particles**
- ◆ **Acid Generation and Neutralization Reactions**

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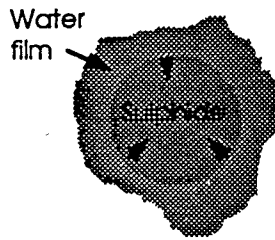
## **ACIDROCK (2)**

- ◆ **Timed Events or Site - Specific Activities**
  - Placement / removal of lifts
  - In-situ leaching
  - Recontouring, cover placement
- ◆ **Equity Silver Waste Rock, In-Situ Uranium Leach Pile, Coal Refuse Piles**

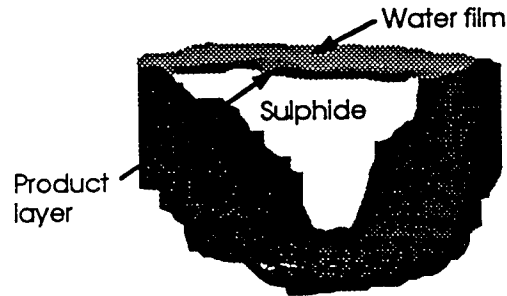
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# Mass Transport Processes

Shrinking Radius Model

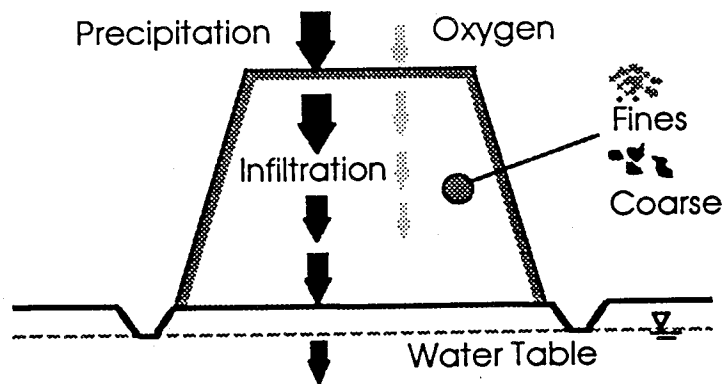


Shrinking Reactive Front Model



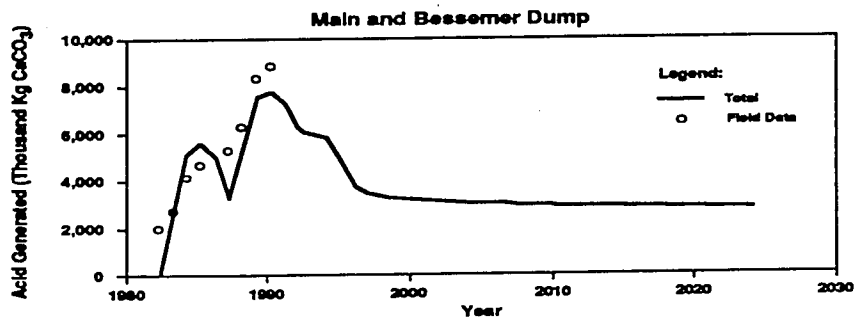
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## Concept for Modelling Waste Rock Heaps / Piles



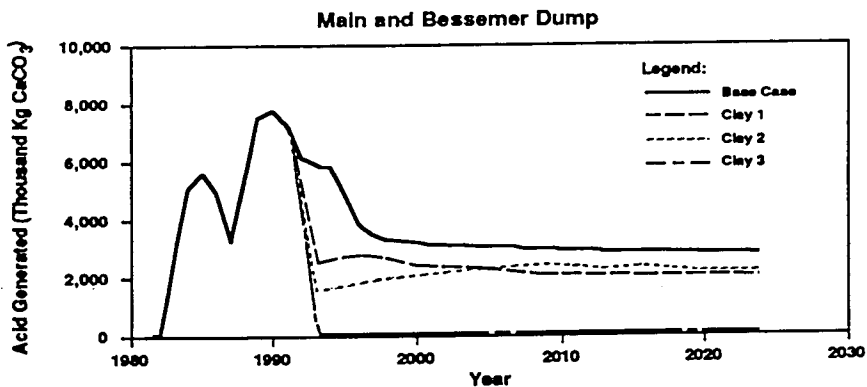
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## Equity Silver Waste Rock Heaps Predicted Acid Generated



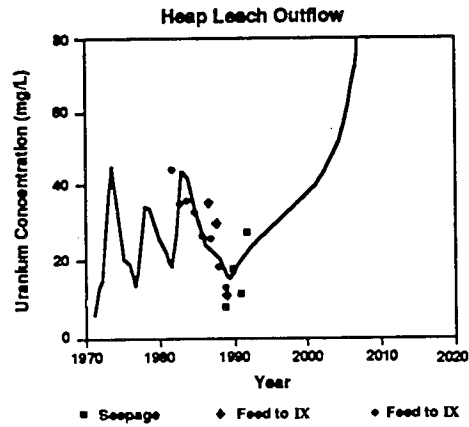
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## Equity Silver Waste Rock Heaps Cover Effectiveness



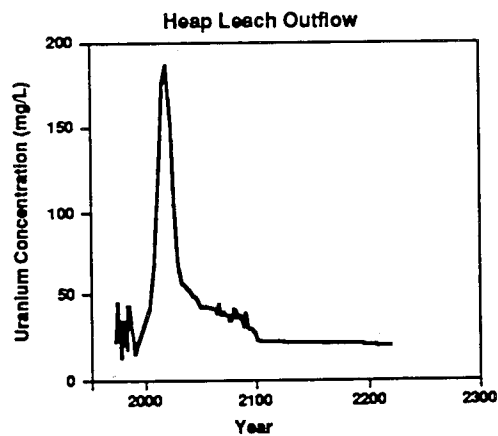
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## Gessenhalde Calibration: Predicted Uranium



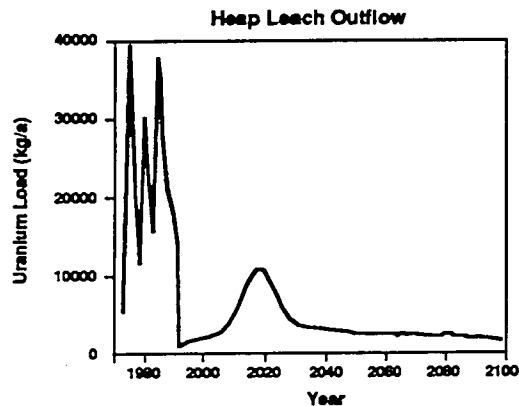
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## Gessenhalde Basecase Scenario: Predicted Uranium



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## Gessenhalde Basecase Scenario: Predicted Uranium Load



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## ROCKSTAR (1)

- ◆ Dynamic Geochemical (computer) Model
- ◆ Multi-Nodal Interconnected Compartments (20)
  - Waste rock piles, tailings, open pit, underground mines, surface water)
  - Each node has up to 20 layers
- ◆ Acid Generation / Neutralization and Release of Contaminants (aqueous species, metals, radionuclides)

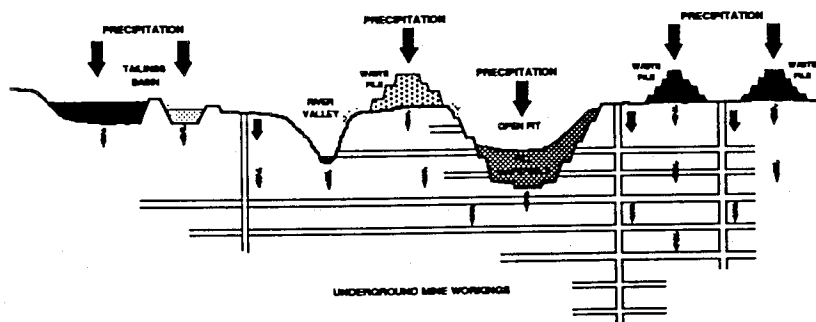
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## ROCKSTAR (2)

- ◆ Simultaneous simulation of 16 Waste Rock Heaps
- ◆ Detailed Simulation of Single Large Heap (10 nodes)
- ◆ Screening Simulation of Flooding of Mining Region (20 nodes)
- ◆ Detailed simulation of Flooding of Mining Region (60 nodes)

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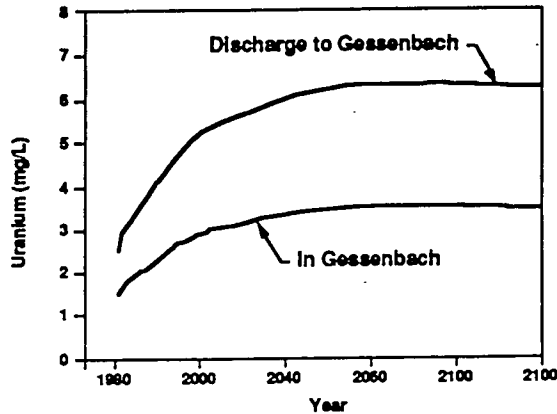
### Schematic of Geochemical Modelling Compartments (Tailings Basins, Waste Piles, Mine, River ... etc.)



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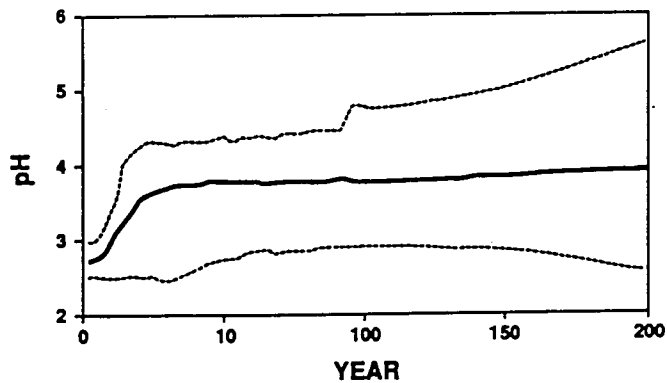


## Gessenhalde Placed in Open Pit Predicted Uranium in River



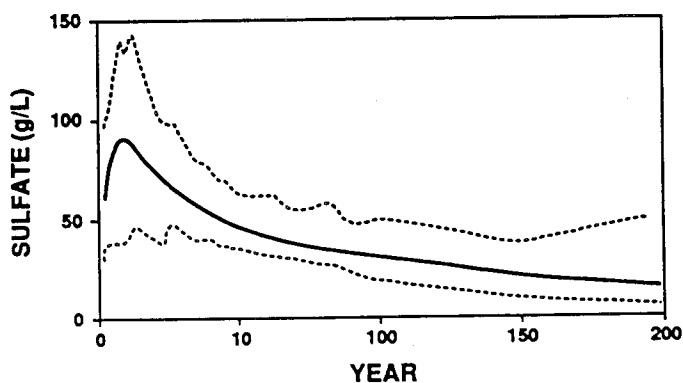
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## Probabilistic Results Large Waste Rock Pile (1)



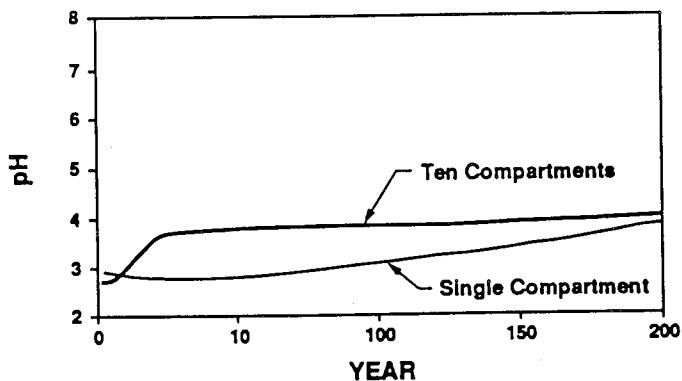
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## Probabilistic Results Large Waste Rock Pile (2)



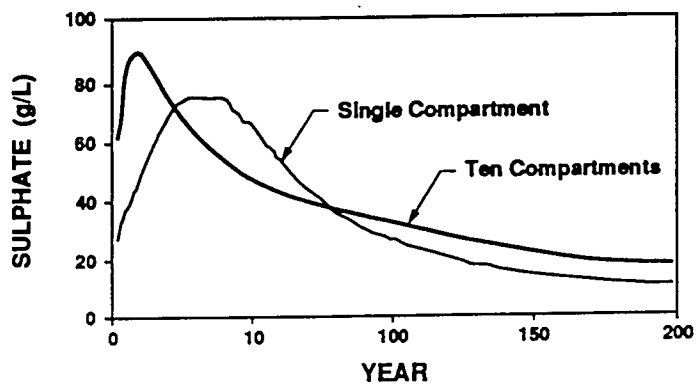
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## Simple Versus Detailed Modelling of Heaps (1)



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## Simple Versus Detailed Modelling of Heaps (2)



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## Conclusions (1)

- ◆ The Usefulness of Mathematical Prediction Models Has Been Demonstrated
  - Better understanding of mechanisms of AMD
  - Comparison of management options

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## **Conclusions (2)**

### **◆ Areas for Improvement**

- Expansion of geochemical capabilities
- Collection of mine site databases
- Improved database of thermodynamic equilibrium constants, sorption coefficients, ....etc.
- Consider kinetic rates for precipitation / dissolution, and surface-controlled reactions
- Apply and develop empirical models

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## **Conclusions (3)**

### **◆ Associated Assessments**

- Management options for mine water
- Evaluation / comparison of treatment alternatives
- Environmental pathways
- Screening level and detailed risk assessments (human, ecological)
- Monitoring program(s)
- Decommissioning plan(s)

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### **3.3. REVIEW OF GEOCHEMICAL MODELS**

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